STABILITY OF SOME PERIODIC ORBITS IN AXIALLY SYMMETRICAL POTENTIAL FIELD

V.A.Antonov, E.I.Timoshkova
Institute of Theoretical Astronomy RAS
Kutuzov Quay 10, 191187 St.Petersburg, Russia

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It is known that the exact circular solution in the equatorial plane exists for the problem of the motion of unity mass in axially symmetrical potential field $V(r,z)$. The perturbed motion in the meridional plane is described by hamiltonian system with two–dimensional polinomial potential. Hamiltonian systems with polinomial potentials are extensively studied up to now. However the problem of the change of motion topology is not yet understood in detail. Formerly we found the full list of the trajectories which are represented through the first and second order algebraic curves for systems with Henon-Heiles potential

$$U = 0.5(x^2 + \lambda z^2) + \alpha x^3 + xz^2$$

Here x, z are the small variations of cylindrical coordinates r, z from values $r = const, z = o$ for circular solution, α, λ are arbitrary parameters. In this paper we examine some from algebraic trajectories for the purpose to reveal their stability in the linear approximation. The problem of orbit stability is solved by means of analysis for differential equation with periodic coefficients. The regions of stable and unstable motions in space of parameters λ, h (h is the energy constant) are found for circular orbits ($\alpha = 1$) by analytical methods. For x–axis orbits the analysis of variation equation in analitical form is feasible only for a sequance

$$\alpha = \frac{4}{n(n+1)}, \quad n = 1, 2, 3.$$

In these cases the dependence for critical amplitude of oscillations on the parameter λ was determined. The instants when the stability replaces by the instability correspond to topological rebuilding of whole complex of orbits which are near to periodicity.