A numerical scheme to integrate the rotational motion of a quasi-rigid body and its application to the Earth rotation

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A new numerical scheme to integrate the orientation of a quasi-rigid body is presented. The basic variables are

$$(L, L_{\rm X}, L_{\rm Y}, L_{\rm A}, L_{\rm B}, f)$$

the magnitude and the X-, Y-, A-, and B-components of the spin angular momentum vector $\mathbf{L}$, and the longitude of A-axis measured from X-axis along the great circle perpendicular to $\mathbf{L}$. The orientation matrix is generated by the five successive rotations in the sense of 1-2-3-2-1 as

$$(\mathbf{e}_{\rm A}, \mathbf{e}_{\rm B}, \mathbf{e}_{\rm C}) = \mathcal{R}_1(\sigma)\mathcal{R}_2(-\varphi)\mathcal{R}_3(-f)\mathcal{R}_2(\beta)\mathcal{R}_1(-\xi)$$

where

$$\sigma = \tan^{-1}\left(\frac{L_{\rm Y}}{\sqrt{L^2 - (L_{\rm X}^2 + L_{\rm Y}^2)}}\right), \qquad \varphi = \sin^{-1}\frac{L_{\rm X}}{L},$$

$$\beta = \sin^{-1}\frac{L_{\rm A}}{L}, \qquad \xi = \tan^{-1}\left(\frac{L_{\rm B}}{\sqrt{L^2 - (L_{\rm A}^2 + L_{\rm B}^2)}}\right).$$

Not all these basic variables but the corrections to their nominal linear motions for some components instead such as

$$(\Delta L, \Delta L_{\rm X}, \Delta L_{\rm Y}, L_{\rm A}, L_{\rm B}, \Delta f)$$

are actually integrated in the inertial coordinate system whose Z-axis is chosen to be close to $\mathbf{L}$ at the initial epoch of integration. These small variables directly correspond to the variation of LOD, the nutations in obliquity and in declination, the polar motion, and the variation of UT1 in terms of the Earth rotation. Numerical simulations showed that the new scheme integrates the Earth orientation matrix 6-7 digits more precisely than the ordinary Eulerian approach does while the required computational time does not change significantly.