ON TWO–PARAMETER LINEARIZING TRANSFORMATIONS FOR UNIFORM TREATMENT OF TWO–BODY MOTION.

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Abstract

The ideas of regularization and linearization of the differential equations governing perturbed dynamical systems have inspired the introduction of adequate transformations of dependent and/or independent variables. As well known, these processes of regularization and linearization play an important role in Celestial Mechanics. In this respect, the study of perturbed Keplerian systems constitutes a field in which these techniques have been put into action. A classical, general reference for this subject is Stiefel & Scheifele (1971).

By generalizing previous research by Belen'kii (1981) and Cid, Ferrer & Elipe (1983), and within the framework of the elliptic–type two–body motion, Ferrándiz & Ferrer (1986) and Ferrándiz, Ferrer & Sein–Echaluce (1987) replaced the physical time by a fictitious time (in fact, a family of them) as the new independent variable for the integration of perturbed Kepler problems. They used the method of linearization by means of time transformations on introducing a family of generalized anomalies defined via a differential relation in which the linearizing function depends on two homogeneous parameters that can be taken as functions of some orbital elements. In the case of pure Kepler problems, they analytically integrated the respective time transformations, in closed form, by means of elliptic integrals and functions. Their developments were originally intended to facilitate (on the basis of some radial intermediaries) the analytical treatment of artificial Earth satellite orbits under a zonal model of geopotential, and as a preconditioning of the problem prior to numerical integration.

The present research deals with the analytical investigation of an appropriate modification and a systematic study of these generalized Sundman–type transformations allowing the extension of the results established by the said authors to a uniform treatment of two–body orbits under a *universal formulation* and representation of the two–body problem, in the sense of Stiefel & Scheifele (1971), §11, or Battin (1987), §4.5 and §4.6. To this end, an essential role is played by certain families of special functions, the so–called *universal*

functions (Battin 1987, §4.5 and §4.6) and the *Stumpff c–functions* (Stiefel & Scheifele 1971, §11). As expected, throughout the required reckoning work when integrating the changes of time parameter under consideration, elliptic integrals and functions will also play an important role. Symbolic computation will allow us to develop and systematize our approach when perturbed motion is analyzed.

KEY WORDS: reparametrizing transformations, generalized anomalies, uniform treatment of two–body motion, universal functions, elliptic integrals and functions.

*** Symposium Topic.** Mathematical Techniques: elliptic functions and algebraic manipulation.

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