D'Alembertian functions in Celestial Mechanics as a space of holomorph–in–a–bottle functions

K.V. Kholshevnikov

Astronomical Institute, St. Petersburg University,
Stary Peterhof, 198904 St. Petersburg, Russia.
E-mail: kvk@aispbu.spb.su

The well–known D'Alembertian characteristic of disturbing and other important for Celestial Mechanics functions means a property of Fourier series coefficients, permitting to replace it by a Maclaurin series in terms of $x = re^{i\phi}, y = re^{-i\phi}, \phi$ being an angular variable, r — action one (e.g. eccentricity or inclination). *We show that such a coefficients property is a consequence of a functional one.* Namely, introduce a notion of D'Alembertian functions which are holomorph and limited in a domain $0 < |r|e^{|Im\phi|} < R$ and are invariant under the transformation $r, \phi \rightarrow -r, \phi + \pi$. Such a domain may be represented like a bottle with infinitely long and infinitely thin neck.

Of course a D'Alembertian function possesses a D'Alembertian characteristic, moreover we can estimate the coefficients, the size of convergence domain and the function itself.

Simpliest example. Consider Cartesian coordinates of a planet in elliptic motion. Let semi–major axis is arbitrary, mean longitude varies in a $2c$ wide strip symmetric with respect to real axis. Then the coordinates are D'Alembertian functions with respect to two pairs: sine of semi–inclination — longitude of ascending node; eccentricity — longitude of pericentre. In the first case $R = 1$, in the second $R = 1/ch\,(c+1)$. Considering only real values we may put $c = 0$, so $R = 0.648$ that is a little bit less than a famous *Laplace limit* 0.663. To reach it we ought to introduce a complicated asymmetrical bottle instead of a simple one above, that appears to be unnecessary .