On the convergence of analytical theories for low satellites

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We are developing fully analytical perturbation theories to compute proper elements for low-altitude satellites. We used a mixed Lagrangian-Hamiltonian formalism, and separately accounted for the very short periodic perturbations (associated with the revolution of the satellite around the central body), the medium periodic perturbations (associated with the rotation of the central body), and the long periodic ones. The theory takes a much simpler form if it is possible to neglect the resonances between short and medium periodic terms, as it is the case for a slowly rotating central body, such as the Moon. For a low satellite, truncation with respect to the degree in eccentricity is legitimate, while the inclination functions have to be handled in full generality.

Even within these assumptions, and having as goal a moderate accuracy (e.g. 0.001 in eccentricity), a simple first order theory is not enough. The two main problems are the slow convergence with respect to the degree and order of the spherical harmonics expansion of the gravity field of the central body, and the presence of second order effects (containing the square of the gravitational potential coefficients). An additional problem arises when the initial conditions correspond to zero eccentricity; in this case –which frequently occurs in mission analysis studies– the truncation with respect to the degree in eccentricity and with respect to the second order in the perturbations are mutually inconsistent. We have used a fixed point technique to avoid this inconsistency and to obtain an accurate result. We have also investigated the second order effects by comparing both the analytical and the numerical solutions with a semianalytical solution; in the case of the Moon, the theory needs to be pushed to second order if the orbits have to be analytically predicted over time spans longer than 1 year.

For the cases where the theory is applicable, the results are accurate enough to analytically predict the orbit with a given initial condition; the computation being fully analytic and explicit, it is of course much more efficient than any conceivable numerical technique. The proper elements can also be used to study problems such as orbit maintenance and optimum manoeuver cycles, with significant practical implications.