Integrable and stochastic motions in the N—body problem

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The well–known results of Poincare and Bruns set limits on the global integrability of the N–body problem. But the regions exist with simple structure of trajectories. Using the Picard's iterations technique, one can find large areas, where the N–body problem is integrable in classical sense. These bodies have large relative velocities and the close approaches are absent. The motions near collisions are more complicated and call for a specific technique. Let the planets move in frameworks of the KAM–theory, zero-mass body travels near orbits with collisions. Convenient approximation is point–like action sphere method. The contraction of action sphere to the point formally leads to indeterministic (stochastic) description of motion by means of tree like Markov process. The result of tree properties investigation is the evidence for presence of trajectory passing near every Keplerian elliptic orbit intersecting any noncircular planetary orbit. In the small neighbourhood of the stochastic approximation one can find the exact solution of the equations of motion; the method of such solution construction is elaborated. For example, the restricted planar circular three-body problem has three resonance collision orbits, the ratios of periods are equal to $1/1$, $1/2$, $2/1$. Arbitrary infinite sequence containing three symbols $1/1$, $1/2$, $2/1$ (successive symbols are different) corresponds to point–like action sphere method trajectory. Using Jacobi's integral one can prove the existence of exact solution of the restricted three-body problem, which is close to such arbitrary sequence for all time.