Nonconcervative effects of the numerical modeling of Hamiltonian systems and symplectic integration

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This report devoted to the influence of nonconservative effects of the numerical modeling methods to the topology of phase portraits of systems of Hamiltonian differential equations. Also we study exact symplectic methods for numerical integration of Hamiltonian systems.

Usually process of modeling of hamiltonian systems is based on some methods of numerical integration of ODE . Orthodox methods like RK methods are not symplectic therefore they do not conserve special properties of Hamiltonian systems like volume preservation. The application of orthodox methods of modeling leads to global changes of topology of the phase portrait. For example attractors can appear. This effect usually appears at integration over a very long time span, as the accuracy of calculations of individual trajectories can be very high. Not long ago exact symplectic methods for numerical integration of Hamiltonian differential equations appeared [1]. They do not bring nonconservative perturbations into the modeled systems. Therefore they keep the properties of the phase portrait, which are stable relative to the small conservative perturbations.

We present a scheme to study of nonconservative effects of orthodox numerical modeling methods, which is based on applications of computer algebra. We construct for a concrete Hamiltonian the one-step map for the method of integration in exact symbolic form. Jacobian of this map is not equal 1, as the Liouville's theorem does not hold for orthodox methods. Therefore the phase space consists of domains of compression and domains of expansion. The behavior of the families of trajectories of dynamical systems, which fall in the one from these domains, strongly changes at the process of

modeling. The analytical calculations of the Jacobian of the one-step map can be made by means of computer algebra system like Reduce and Maple. in analytical form too. Solving the equation $\det(J) = 1$ we get a manifold, which separates the domains of expansion from ones of compression. As an example the restricted three body problem and the Henon-Heiles Hamiltonian of two degrees of freedom were considered. To get the phase portrait a Poincare map was constructed. We studied the nonconservative effects of the Runge-Kutta method near libration points for restricted three body problem and in all phase space for Henon-Heyles hamiltonian. As energy dissipation in observed we could not limit oneself by investigation of the phase portrait on the standard section of Poincare map only, where E is constant on this standard section. Therefore the phase portraits were presented at other sections, where first coordinate means energy E and second one is a function of coordinates and impulses of the system.

It will be noted that an integration by exact symplectic method allows to keep a topology of the phase portrait almost on the whole phase space. We also present an explicit symplectic scheme for the elliptic restricted three body problem. In this case hamiltonian has not separate form and we can not directly use Yoshida methods, but it can be presented as sum of two integrable hamiltonians. Beside this we present some results about estimation of errors of action variables for symplectic methods. These estimations are based on KAM theory. At the end we discuss some applications of manipulating with formal Lie series to the problem of constructing symplectic integrators.

References

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