EPHEMERIDES
OF CHAOTIC PLANETS

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Invited review, IAU Symposium 172, Paris, July 1995

We have now a good understanding of the properites of the solutions of the N–body problem over a very long time span, both in the ordered case of conditionally periodic orbits and in the chaotic case, with its homoclinic/heteroclinic tangle. However, when such a conceptual framework is applied to the orbits of real solar system objects, a paradox arises. On one hand, essentially all the orbits, when computed within a realistic dynamical model, turn out to be chaotic. On the other hand, most solar system objects are dynamically very old, and indeed the orbits can be computed both analytically and numerically. Thus celestial mechanicians spend most of their time computing ephemerides of chaotic planets. This is of course possible over time spans either shorter than, or comparable to, the Lyapounov time. Over a much longer timespan, it is not clear why such computations could give useful results.

I will propose a possible way out of this paradox. There are chaotic orbits for which a strong chaotic wandering occurs along only some directions in the phase space. If the coupling among different degrees of freedom is either weak or highly asymmetric, then a random walk in the value of one action variable, and complete randomisation in one angle variable, can occur together with a very slow diffusion in the direction of the other action variables. This turns out to be the case for the coupling of secular perturbations with mean motion resonances of moderate to high order. Thus a random walk in the mean longitude, which occurs for essentially all the planetary orbits, does not imply a significant diffusion in the proper eccentricity and inclination. For this reason the proper elements are constant –to a very good accuracy– for most main belt and trojan asteroids, over timespans longer than the Lyapounov times by three orders of magnitude. On the contrary, when chaos occurs in the secular part of the equations of motion, large instabilities can occur for supposedly main belt asteroids and even for the major planets. Thus ephemerides of chaotic planets can be reliably computed, provided only some of the elements are described by the corresponding mean elements; this is possible in most cases, but by no means in all cases.